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Title: Applications of multichannel R-matrix theory to light nuclei

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Applications of multichannel R-matrix theory to light nuclei

G. M. Hale and M. W. Paris

T-2, LANL

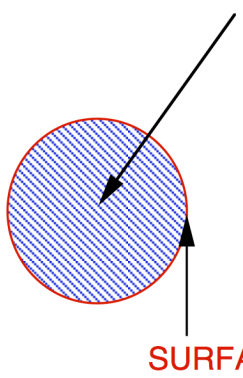
NNSA National Nuclear Security Administration

Outline

- Introduction to R-matrix theory, Bloch operator
- The LANL Energy Dependent Analysis code
- Wigner/Eisenbud and Kapur/Peierls boundary conditions
- R-matrix expansions in the complex energy plane, T-matrix poles and resonance parameters
- Examples: analysis of reactions in the *NN* and ^{5}He systems
- Approximate description of 3-body final states (ex: t+t n-spect.)
- Unphysical extension of the theory to zero channel radii; connection with EFT (exs. n-p scattering, d+t reaction)
- Summary, conclusions

Schematic of R-matrix Theory

INTERIOR (Many-Body) REGION
(Microscopic Calculations)



$$|\psi^+) = (H + \mathcal{L}_B - E)^{-1} \mathcal{L}_B |\psi^+)$$

$$\mathcal{L}_B = \sum_c |c)(c| \left(\frac{\partial}{\partial r_c} r_c - B_c \right),$$

$$(\mathbf{r}_c|c) = \frac{\hbar}{\sqrt{2\mu_c a_c}} \frac{\delta(r_c - a_c)}{r_c} [(\phi_{s_1}^{\mu_1} \otimes \phi_{s_2}^{\mu_2})_s^{\mu} \otimes Y_l^m(\hat{\mathbf{r}}_c)]_J^M$$

$$R_{c'c} = (c'|(H + \mathcal{L}_B - E)^{-1}|c) = \sum_\lambda \frac{(c'|\lambda)(\lambda|c)}{E_\lambda - E}$$

ASYMPTOTIC REGION
(S-matrix, phase shifts, etc.)

$$(r_{c'}|\psi_c^+\rangle = -I_{c'}(r_{c'})\delta_{c'c} + O_{c'}(r_{c'})S_{c'c}$$

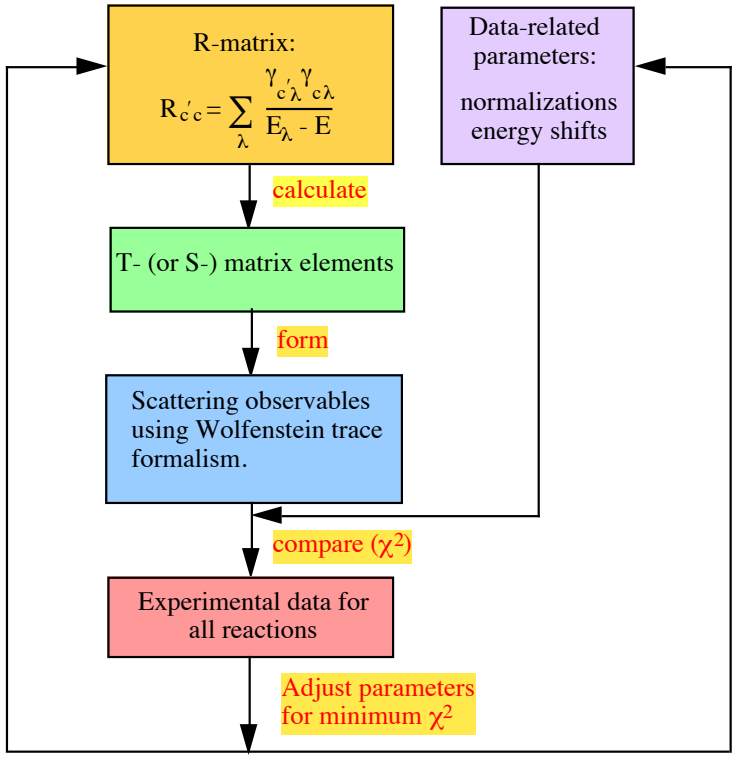
or equivalently,

$$(r_{c'}|\psi_c^+\rangle = F_{c'}(r_{c'})\delta_{c'c} + O_{c'}(r_{c'})T_{c'c}$$

Measurements

Some properties of EDA:

Energy Dependent Analysis Code



- Accommodates general (spins, masses, charges) two-body channels
- Uses relativistic kinematics and R-matrix formulation
- Calculates general scattering observables for 2 ➔2 processes
- Has rather general data-handling capabilities (normalizations, energy shifts, resolution folding)
- Uses modified variable-metric search algorithm that gives parameter covariances at a solution

EDA Analyses of Light Systems

A	System	Channels	Energy Range (MeV)
2	N-N	p+p; n+p, γ+d	0-30 0-40
3	N-d	p+d; n+d	0-4
4	^{4}H ^{4}Li	n+t p+^{3}He	0-20
	^{4}He	p+t n+^{3}He d+d	0-11 0-10 0-10
5	^{5}He	n+α d+t ^{5}He+γ	0-28 0-10
	^{5}Li	p+α d+^{3}He	0-24 0-1.4

A	System (Channels)
6	^{6}He (^{5}He+n, t+t); ^{6}Li (d+^{4}He, t+^{3}He); ^{6}Be (^{5}Li+p, ^{3}He+^{3}He)
7	^{7}Li (t+^{4}He, n+^{6}Li); ^{7}Be (γ+^{7}Be, ^{3}He+^{4}He, p+^{6}Li)
8	^{8}Be (^{4}He+^{4}He, p+^{7}Li, n+^{7}Be, p+^{7}Li*, n+^{7}Be*, d+^{6}Li)
9	^{9}Be (^{8}Be+n, d+^{7}Li, t+^{6}Li); ^{9}B (γ+^{9}B, ^{8}Be+p, d+^{7}Be, ^{3}He+^{6}Li)
10	^{10}Be (n+^{9}Be, ^{6}He+α, ^{8}Be+nn, t+^{7}Li); ^{10}B (α+^{6}Li, p+^{9}Be, ^{3}He+^{7}Li)
11	^{11}B (α+^{7}Li, α+^{7}Li*, ^{8}Be+t, n+^{10}B); ^{11}C (α+^{7}Be, p+^{10}B)
12	^{12}C (^{8}Be+α, p+^{11}B)
13	^{13}C (n+^{12}C, n+^{12}C*)
14	^{14}C (n+^{13}C)
15	^{15}N (p+^{14}C, n+^{14}N, α+^{11}B)
16	^{16}O (γ+^{16}O, α+^{12}C)
17	^{17}O (n+^{16}O, α+^{13}C)
18	^{18}Ne (p+^{17}F, p+^{17}F*, α+^{14}O)

Two Types of R Matrices

W/E boundary conditions:

B_c real, energy-independent.

$$R^B_{c'c} = \sum_\lambda \frac{\gamma_{c'\lambda}\gamma^{\mathrm{T}}_{\lambda c}}{E_\lambda - E}$$

is real, symmetric, TRI, unitary.

K/P boundary conditions:

$$B_c \rightarrow L_c = \frac{r_c}{O_c}\frac{\partial O_c}{\partial r_c}\bigg|_{r_c=a_c}$$

complex, energy-dependent.

$$R^L_{c'c} = \sum_{\lambda'\lambda} \gamma_{c'\lambda'} A_{\lambda'\lambda}(E) \gamma^{\mathrm{T}}_{\lambda c}$$

$$A^{-1}_{\lambda'\lambda}(E) = (\lambda' \mid H + \mathcal{L}_L - E \mid \lambda)$$
$$= (E_\lambda - E)\delta_{\lambda'\lambda} - \gamma^{\mathrm{T}}_{\lambda' c}(L_c - B_c)\gamma_{c\lambda}$$

$\mathbf{R_L} = [\mathbf{1} - \mathbf{R_B}(\mathbf{L} - \mathbf{B})]^{-1}\mathbf{R_B}$ is the outgoing-wave Green's function projected onto the channel surface; its poles are those of the *T*-matrix, given by

$$\mathbf{T} = \mathbf{O}^{-1}\mathbf{R_L}\mathbf{O}^{-1} \qquad \underbrace{-\mathbf{F}\mathbf{O}^{-1}}_{\text{hard-sphere amplitude}} \qquad \left\{ O_c^{-1} = P_c^{\frac{1}{2}} \exp(-i\phi_c) \text{ at real energies} \right.$$

Approach: use $\mathbf{R_B}$ for fitting experimental data and $\mathbf{R_L}$ for interpreting results

Properties of the Two Types of R-matrices

R_B is a meromorphic function with poles only on the real energy axis.

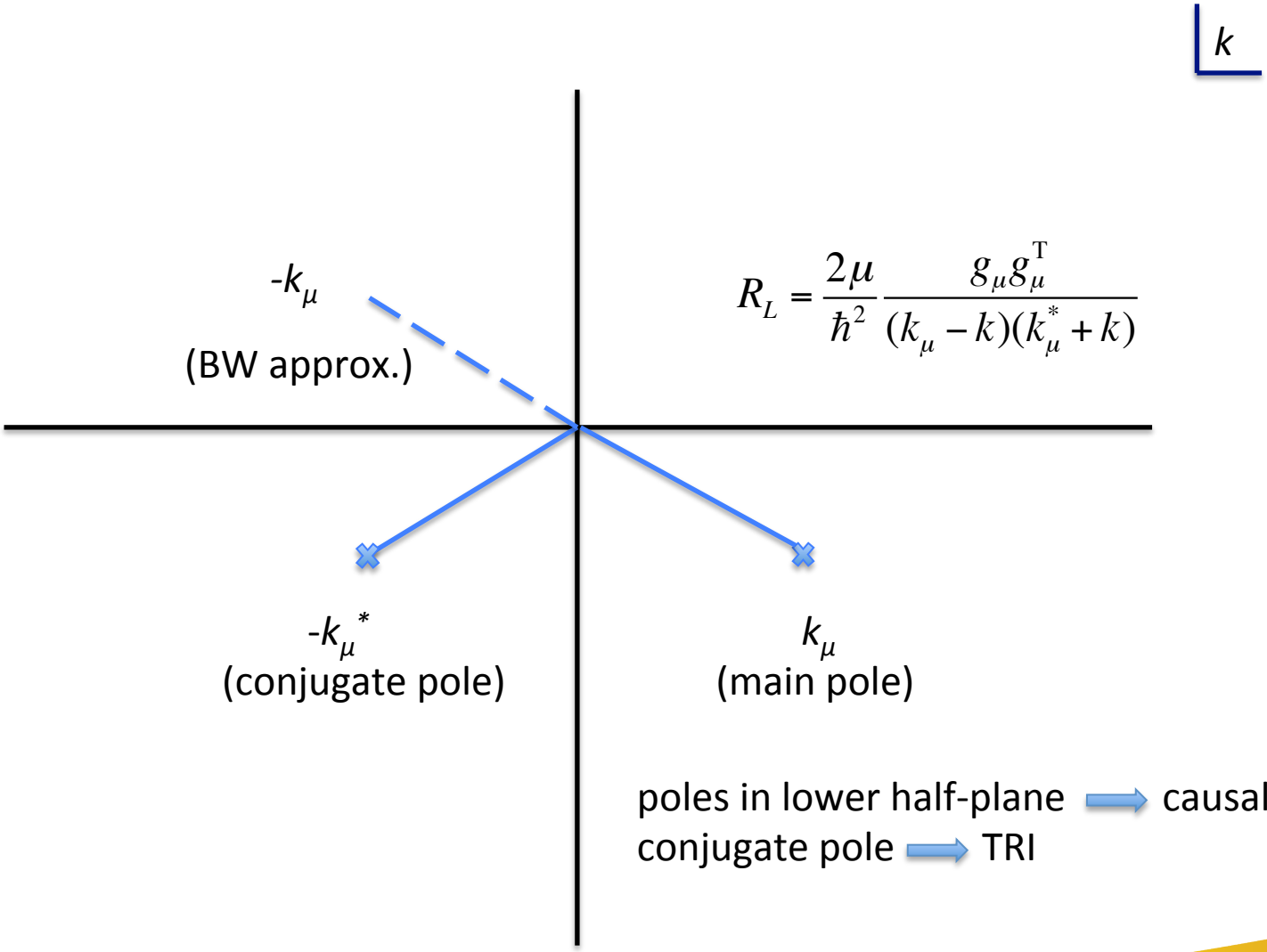
The analytic structure of R_L is more complicated, with poles (in k) in the complex plane, and cuts along the real energy axis.

The eigenfunctions of $H+\mathcal{L}_B$ form a complete, orthogonal set in the internal region. The eigenenergies of the expansion are real.

The eigenfunctions of $H+\mathcal{L}_{L(E)}$ for fixed energy form a complete, bi-orthogonal set in the internal region. The eigenenergies of the expansion are complex. This was the original idea of the Kapur-Peierls expansion, but its parameters are energy-dependent and complex. Using the "true" poles of R_L gets rid of the energy dependence, but the associated eigenfunctions are no longer complete or bi-orthogonal (part of a Berggren basis).

Therefore, R_B has the nicest mathematical properties, but is the hardest to interpret. R_L expansions are much messier, but its poles and residues are directly connected to resonances.

Poles of R_L



poles in lower half-plane ⟹ causality
conjugate pole ⟹ TRI

Resonance Parameters (RP)

Near a pole of *T* at $E = E_\mu$ $(k = k_\mu)$ on an unphysical sheet,

$$\mathbf{T} \approx \mathbf{O}^{-1} \frac{\mathbf{g}_\mu \mathbf{g}_\mu^{\mathrm{T}}}{E_\mu - E} \mathbf{O}^{-1} = \frac{1}{2} \frac{\Gamma_\mu^{\frac{1}{2}} \Gamma_\mu^{\frac{1}{2}\,\mathrm{T}}}{E_\mu - E} \begin{cases} \Gamma_{c\mu} = 2\left|g_{c\mu}\right|^2 \left|O_c(k_\mu)\right|^{-2}, \\ E_\mu = \dfrac{\hbar^2}{2\mu} k_\mu^2 = E_r - \dfrac{1}{2} i\Gamma_\mu \end{cases}$$

However, $\Gamma_\mu = -2\Im E_\mu = 2\sum_c \left|g_{c\mu}\right|^2 \Im L_c(k_\mu) \neq \sum_c \Gamma_{c\mu}$ unless $\Im L_c(k_\mu) = \left|O_c(k_\mu)\right|^{-2}$.

This is true on the real axis, but not in the complex plane. Therefore, define the "strength" of a resonance by

$s = \dfrac{\sum_c \Gamma_{c\mu}}{\Gamma_\mu} \approx 1$ for a narrow resonance.

Radial Independence of the RPs

Note that at the resonance energy, $W(\mathbf{O},\mathbf{U})\big|_{r_c=a_c,E=E_\mu} = \mathbf{O}\mathbf{U}' - \mathbf{O}'\mathbf{U} = 0$, expressing the condition that the radial solution matrix **U** is proportional to the outgoing-wave solutions at the channel surface. This Wronskian condition also holds for all radii $r_c > a_c$, establishing the radial independence of the resonance energy E_μ.

Similarly, the fact that $g_{c\mu} \propto O_c(k_\mu)$ means that $\Gamma_{c\mu} = 2\left|g_{c\mu}\right|^2 \left|O_c(k_\mu)\right|^{-2}$ is also formally independent of channel radius for $r_c > a_c$. Thus, both the partial widths $\Gamma_{c\mu}$ and the total width $\Gamma_\mu = -2\Im E_\mu$ are independent of channel radius, meaning that the strength is also.

The radial independence of these parameters has been observed in practice for the two lowest-lying resonances in ^{5}He ($3/2^-$, $1/2^-$), using the *n*-α *R*-matrix parameters of Barker that were defined at a much larger radius than we used in our ^{5}He analysis.

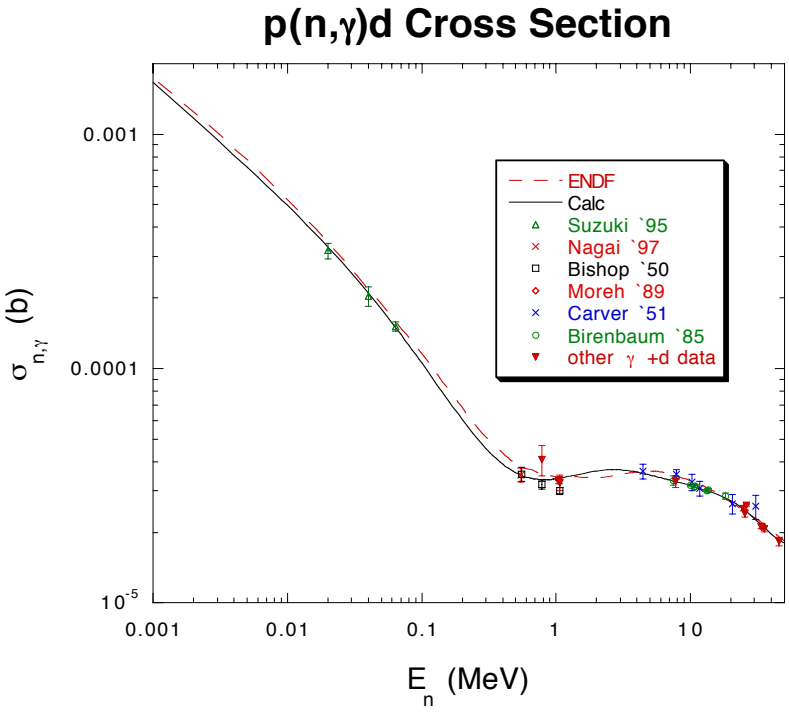
Charge-Independent Analysis of *N-N* Scattering up to 30 MeV

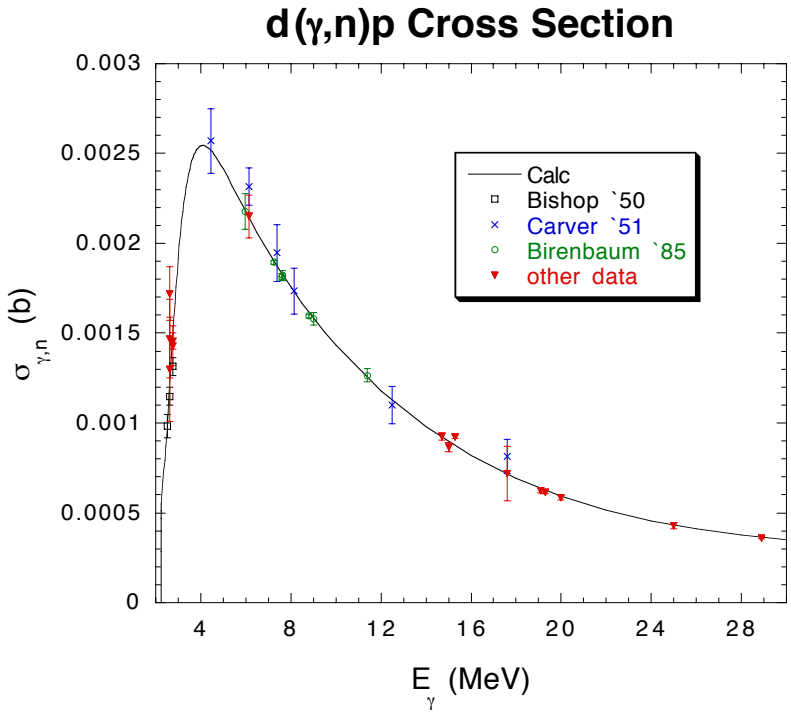
Channel	a_c (fm)	l_{max}
p+p	3.26	3
n+p	3.26	3
γ+d	40	1

Reaction	# Pts.	χ^2	Observable Types
p(p,p)p	692	815	$\sigma(\theta)$, $A_y(p)$, $C_{x,x}$, $C_{y,y}$, $K_x^{x'}$, $K_y^{y'}$, $K_z^{x'}$
p(n,n)p	4378	3232	σ_T, $\sigma(\theta)$, $A_y(n)$, $C_{y,y}$, $K_y^{y'}$
p(n,γ)d	80	133	σ_{int}, $\sigma(\theta)$, $A_y(n)$
d(γ,n)p	59	35	σ_{int}, $\sigma(\theta)$, $\Sigma(\gamma)$, $P_y(n)$
Norms.	129	72	
Total	5338	4287	19

free parameters = 44+129 ⇒ χ^2/degree of freedom = 0.830

n+p Capture and *γ+d* Photodisintegration



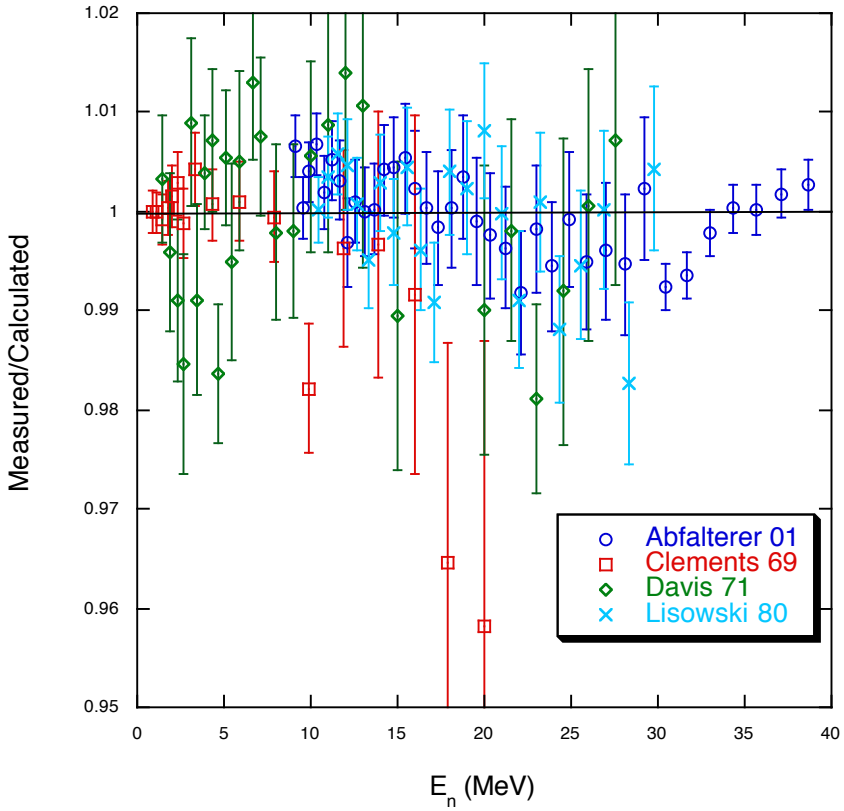


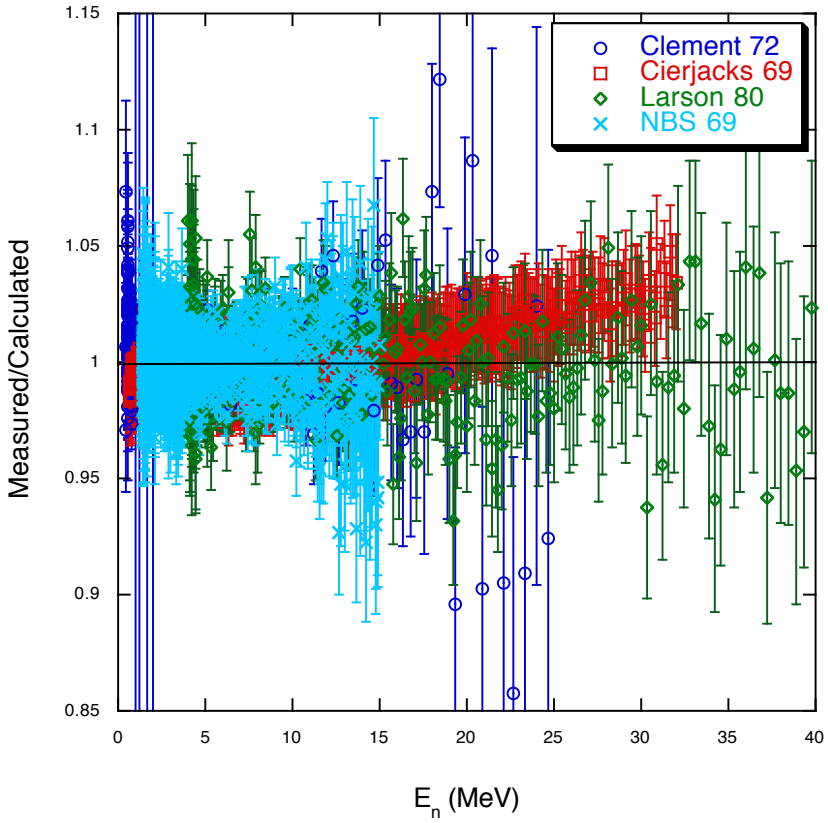
n-p Scattering Lengths

From the analysis,
a_0 = -23.719(5) fm, a_1 = 5.414(1) fm,
giving
$a_c = (3a_1 + a_0)/4$ = -1.8693 fm,
$\sigma_{pol} = (a_1^2 - a_0^2)/4$ = -1.3332 b,
$\sigma_{sc} = \pi(3a_1^2 + a_0^2)$ = 20.437 b.
The first two agree exactly with experimental values, while the last one agrees with the measurement of Houk, (20.436 ± 0.023) b, but not with that of Dilg, (20.491 ± 0.014) b.

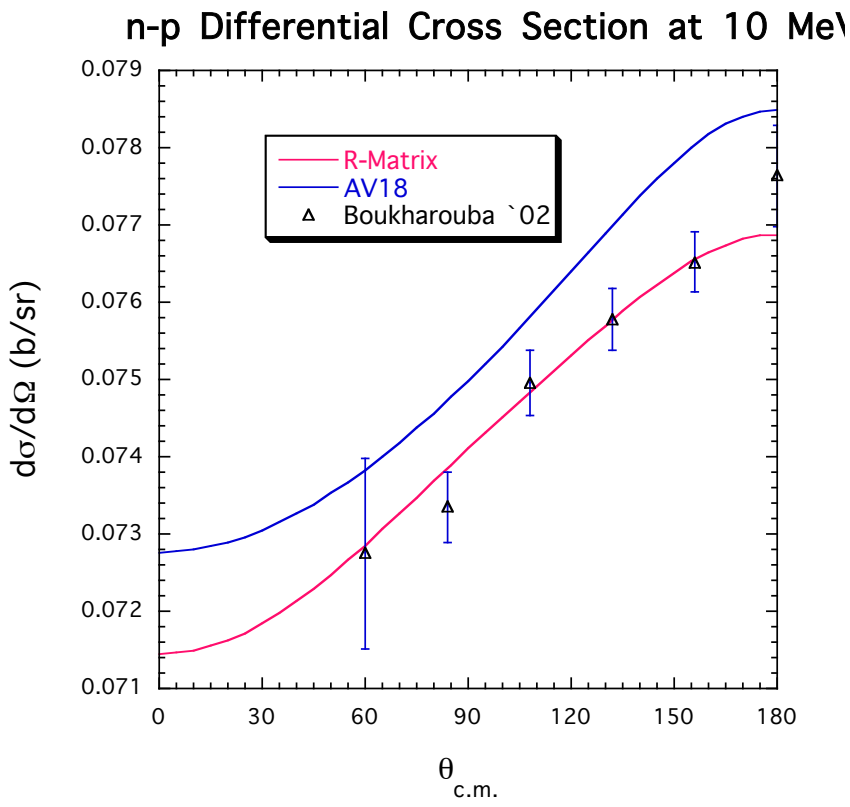
The spin-dependent scattering lengths from AV_{18} are
a_0 = -23.732 fm, a_1 = 5.419 fm,
in good agreement with those from the analysis.

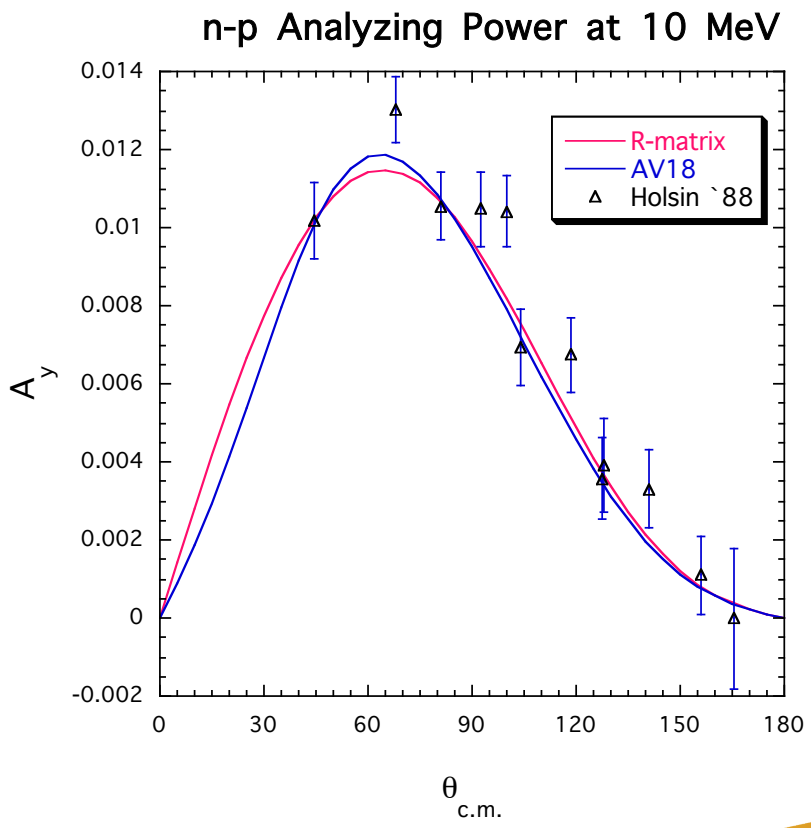
n-p Total Cross Sections



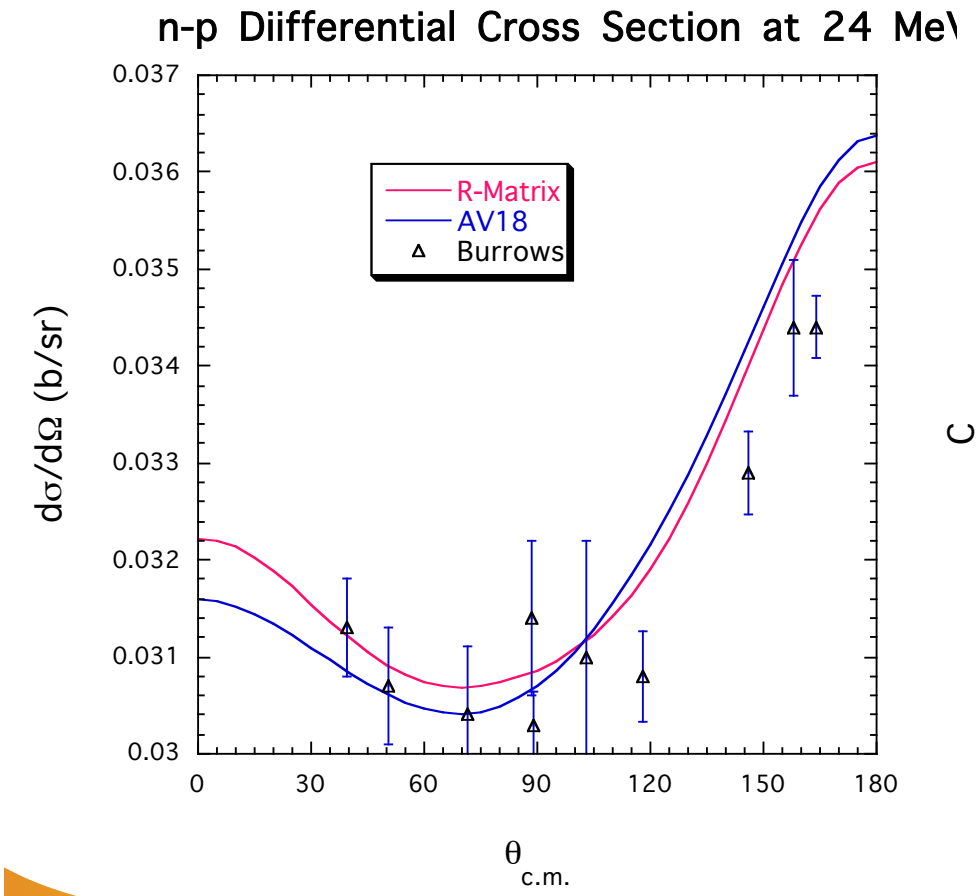


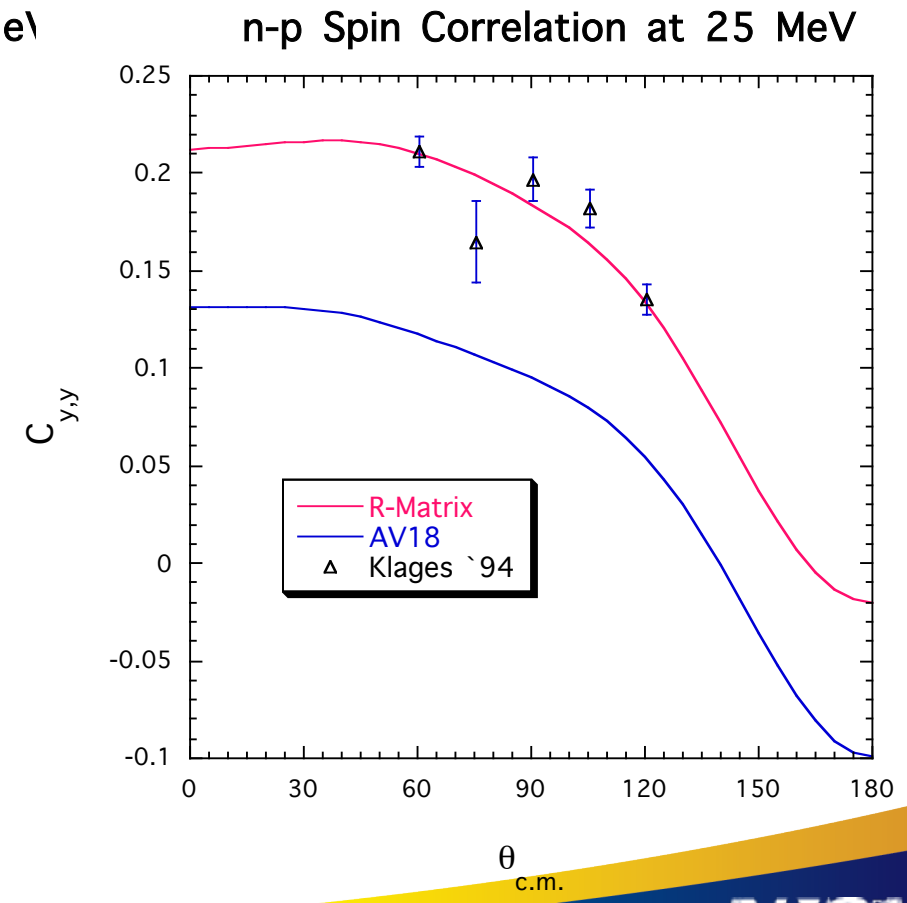
n-p Scattering at 10 MeV





n-p Scattering at ~ 25 MeV



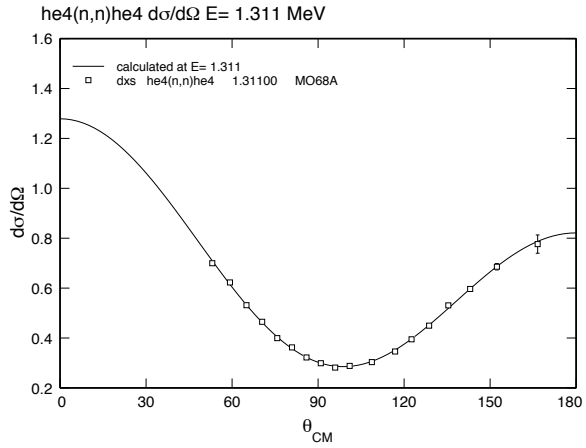


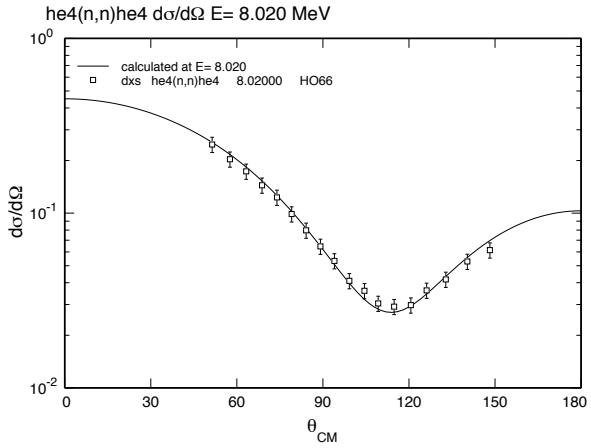
Analysis of Reactions in the ^{5}He System

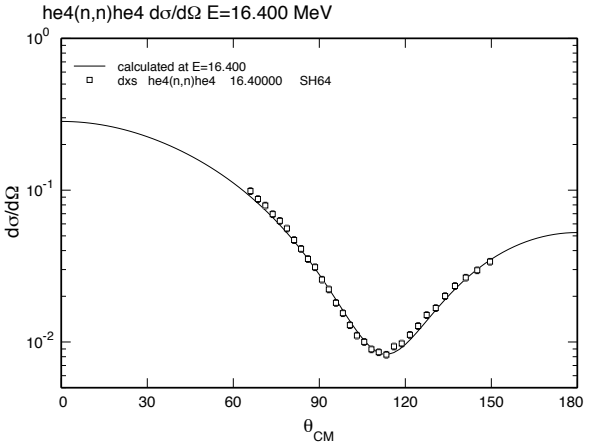
channel	a_c (fm)	l_{max}
n+^{4}He	3.0	5
γ+^{5}He	60	1
d+^{3}H	5.1	5
n+^{4}He*	5.0	1

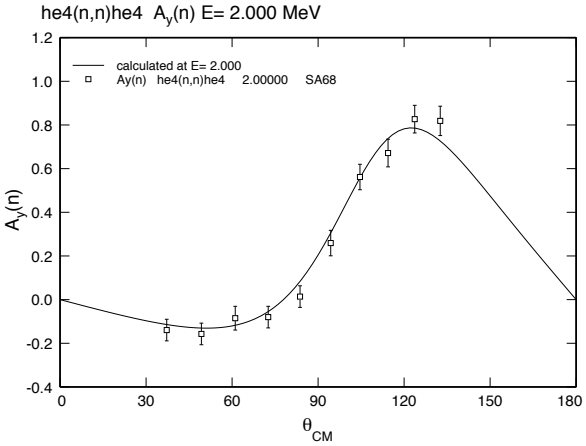
Reaction	Energies (MeV)	# data points	# data types
^{4}He(n,n)^{4}He	E_n = 0 – 28	817	2
^{3}H(d,d)^{3}H	E_d = 0 – 8.6	700	6
^{3}H(d,n)^{4}He	E_d = 0 – 11	1185	14
^{3}H(d,γ)^{5}He	E_d = 0 – 8.6	17	2
^{3}H(d,n)^{4}He*	E_d = 4.8 – 8.3	10	1
total		2729	25

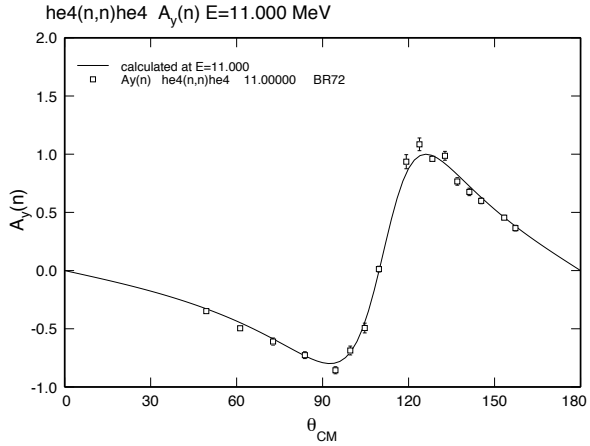
^{4}He(n,n)^{4}He Scattering

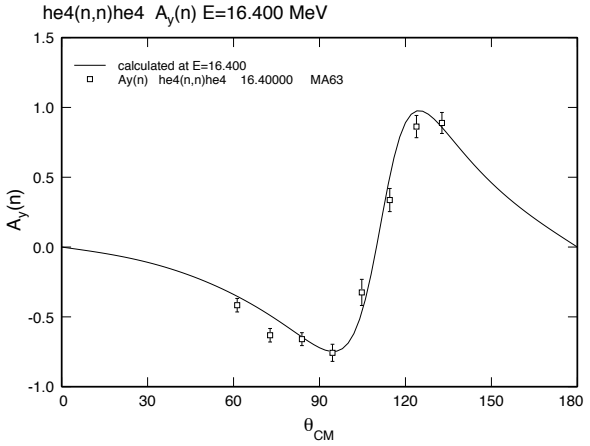




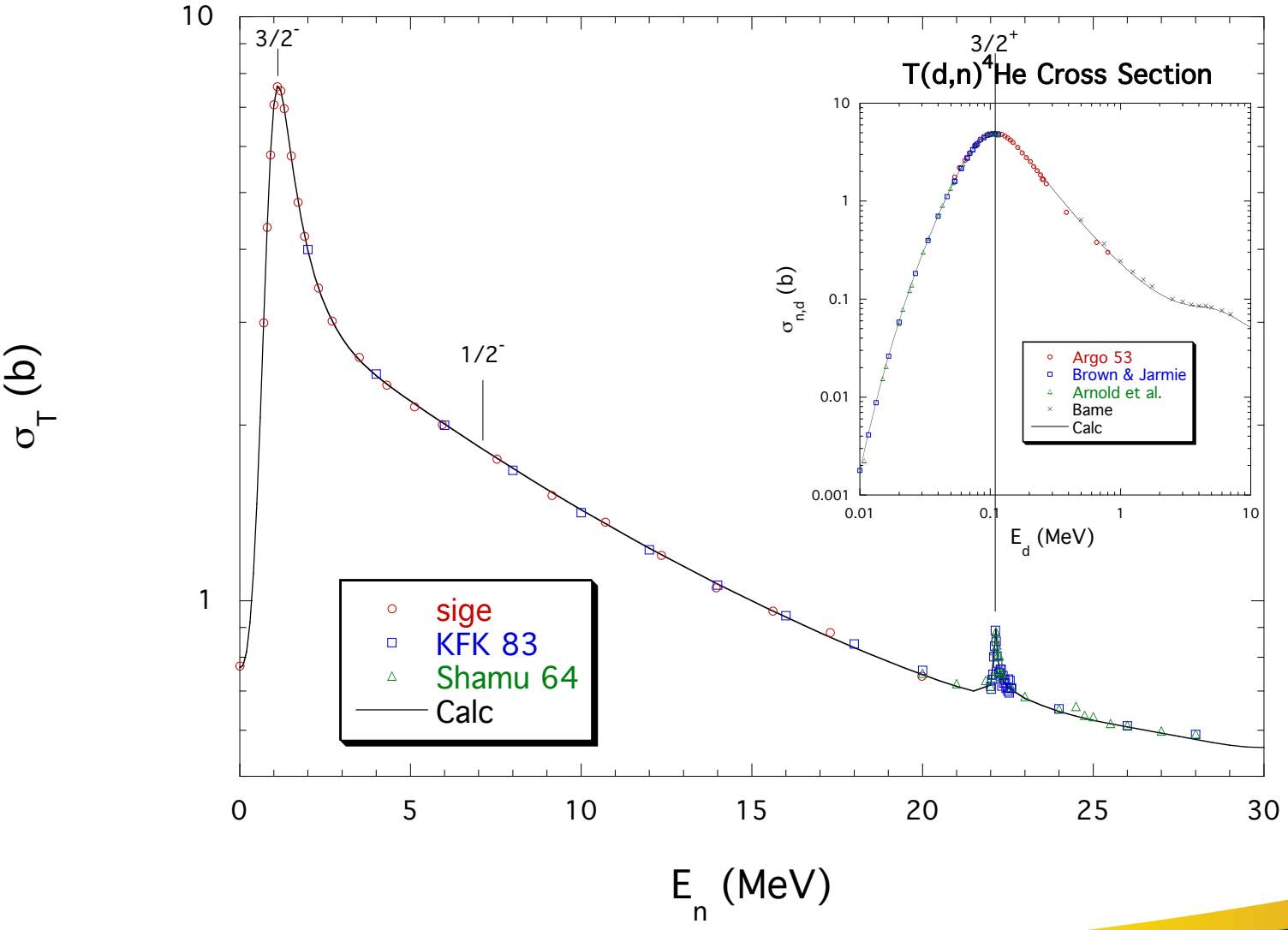




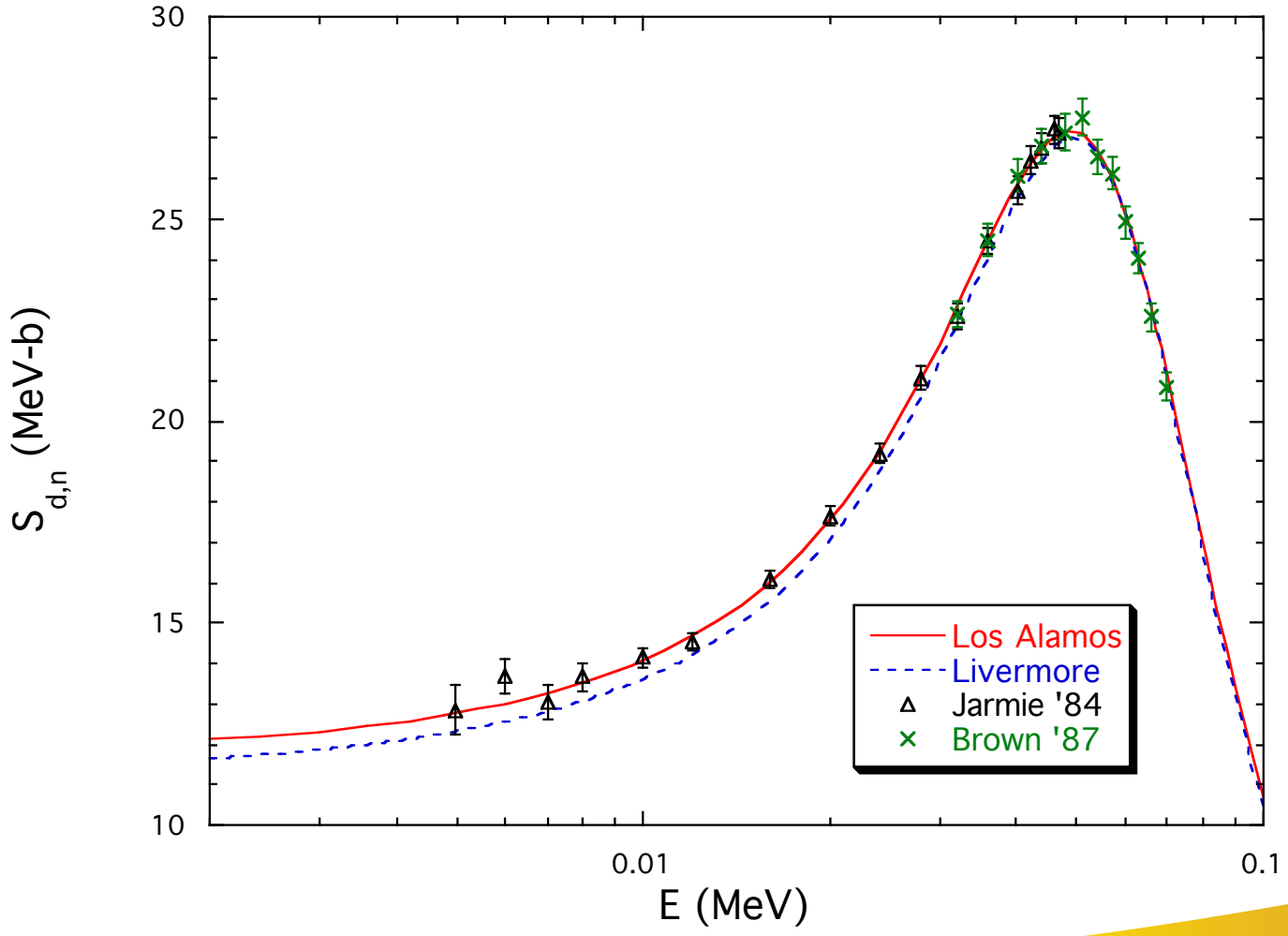




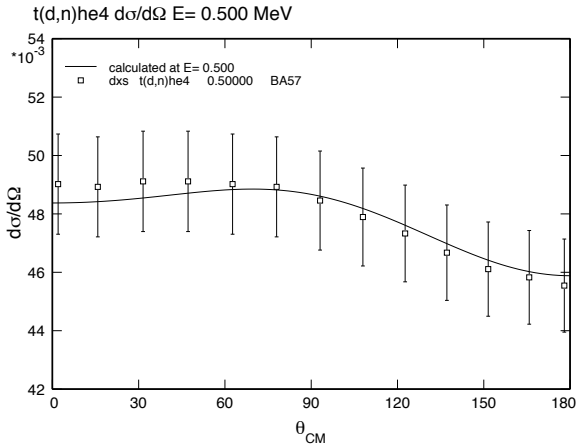
Integrated Cross Sections

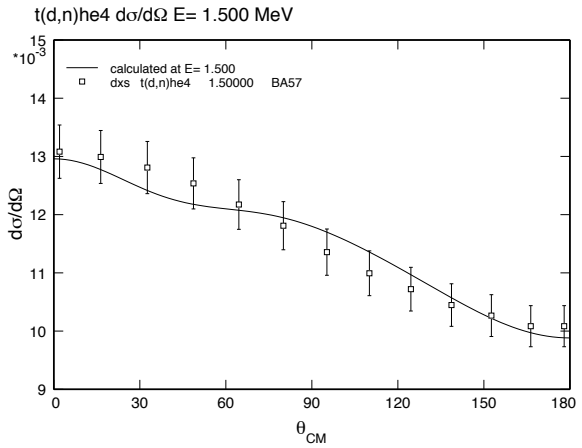


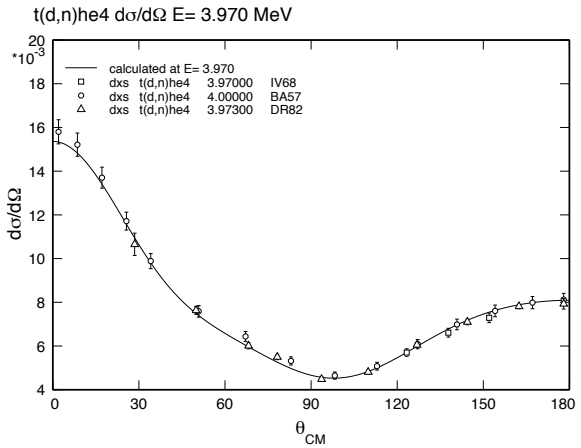
S-factor for T(d,n)^{4}He Reaction

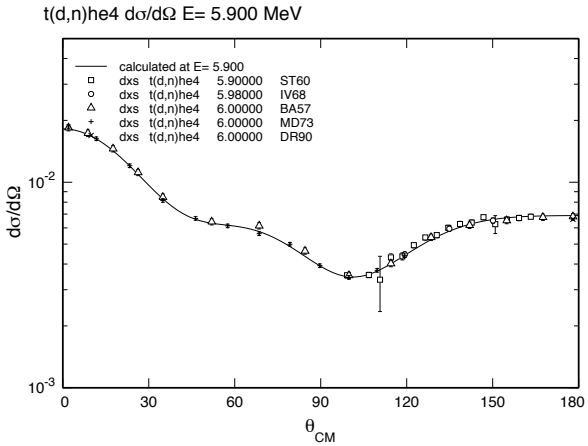


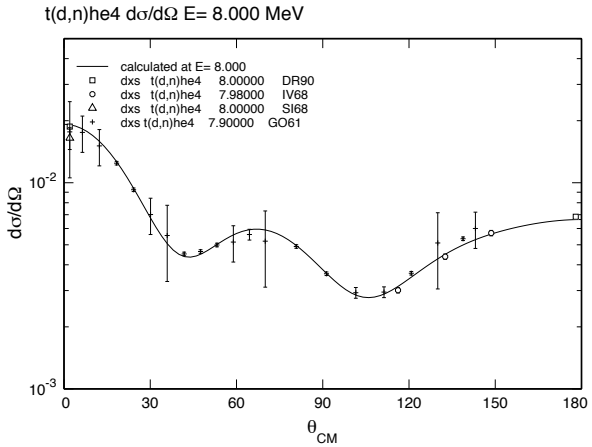
$^{3}H(d,n)^{4}He$ Differential Cross Section

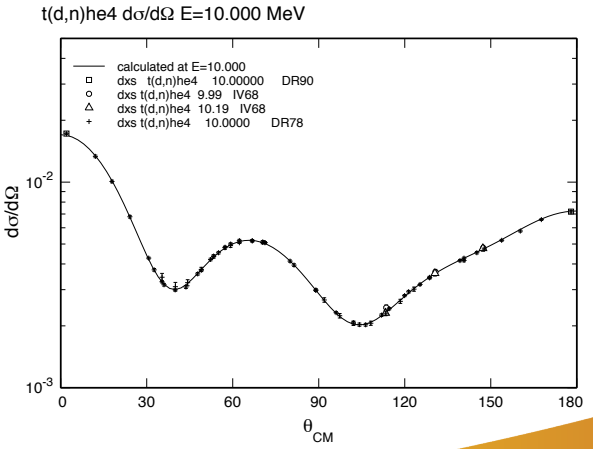




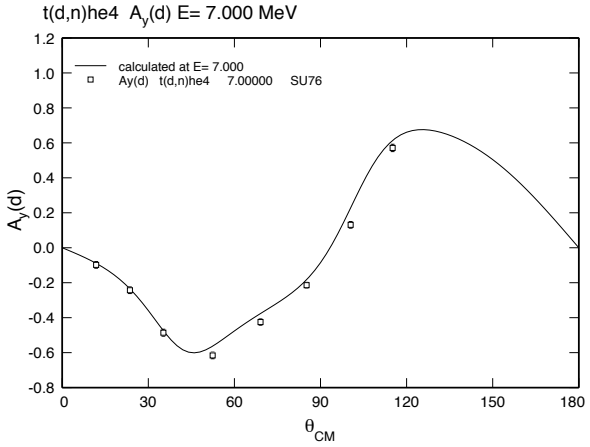


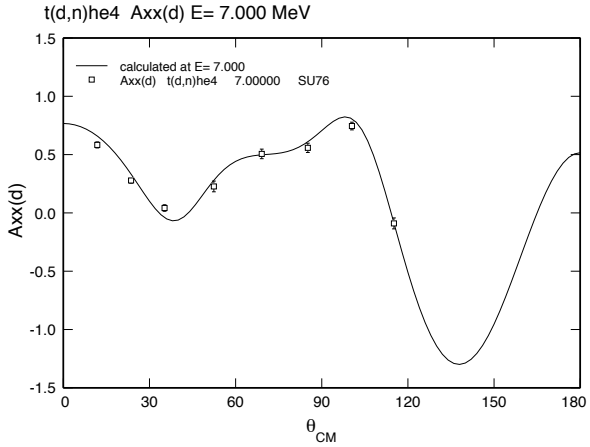


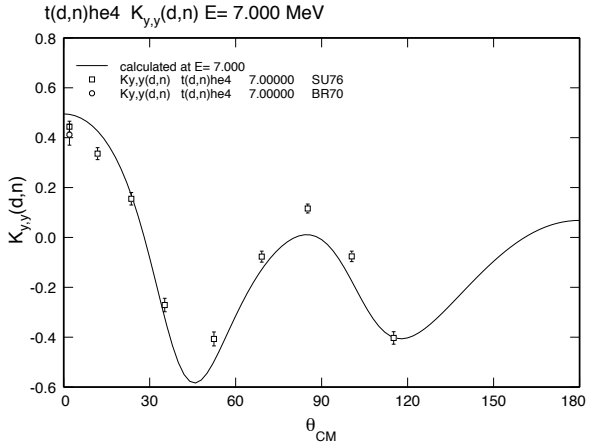


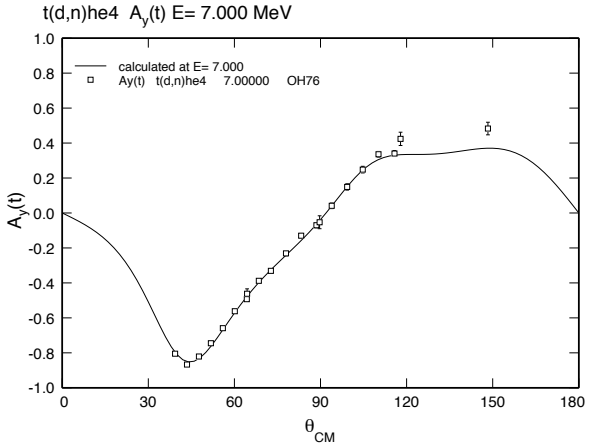


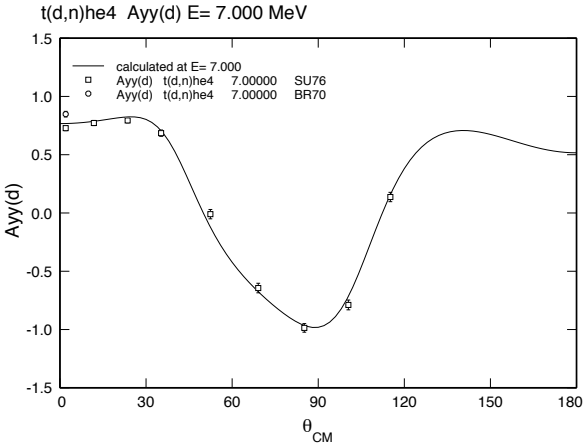
$^{3}H(d,n)^{4}He$ Polarizations at 7 MeV

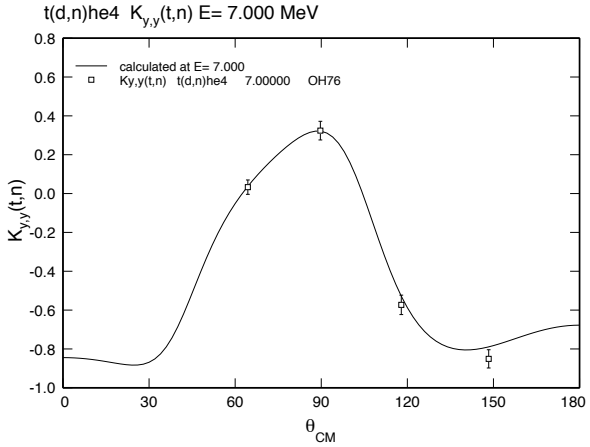




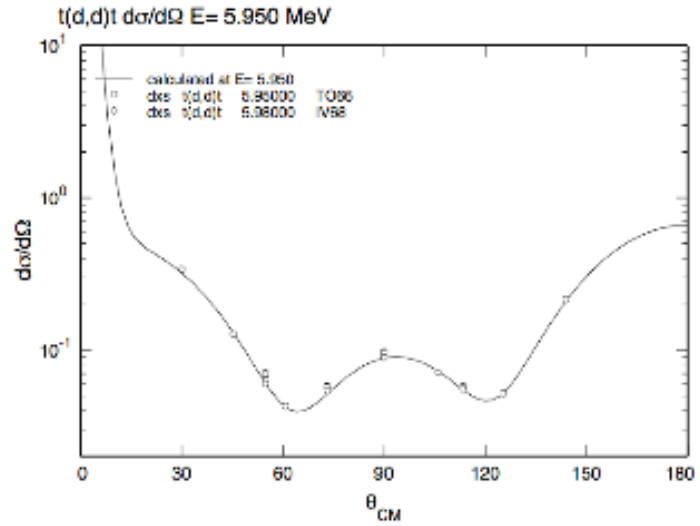


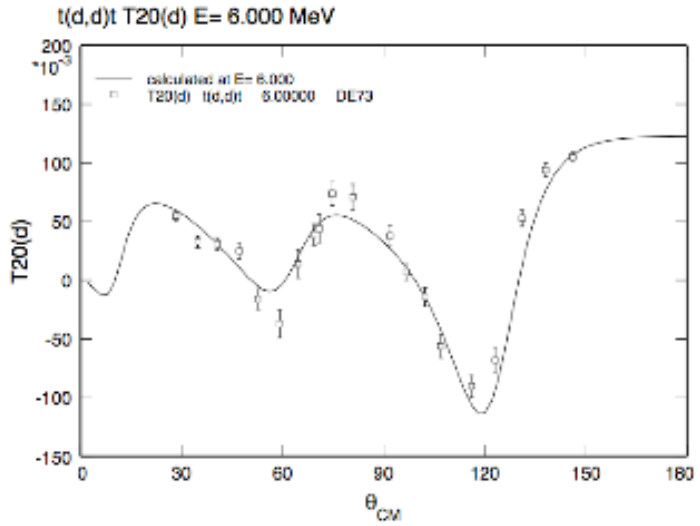


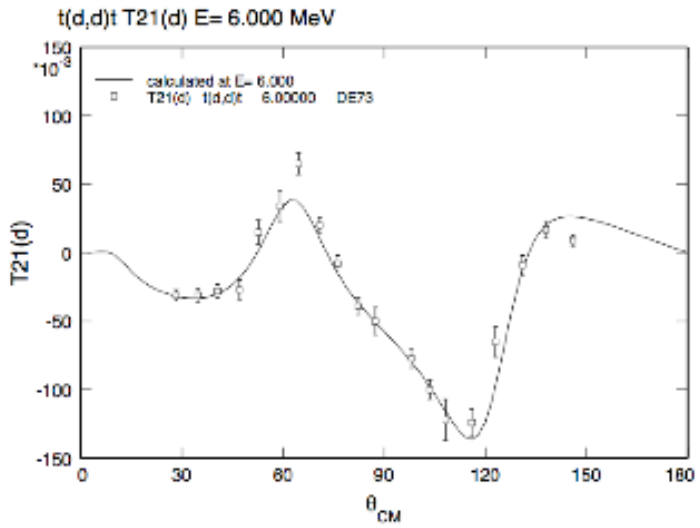


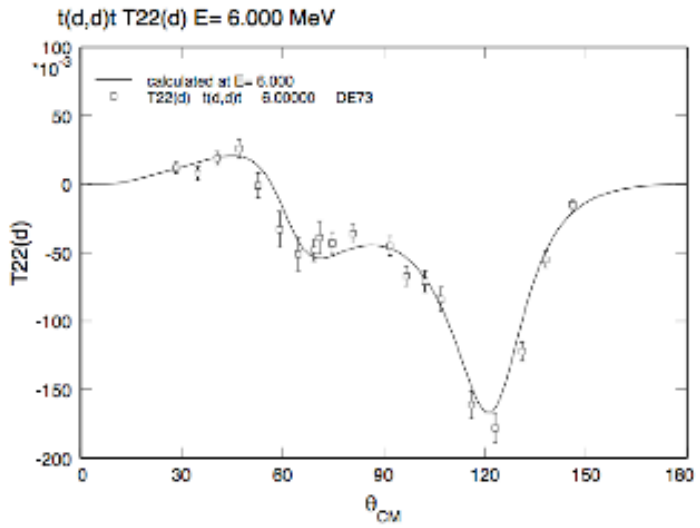


$^3H(d,d)^3H$ Observables at 6 MeV

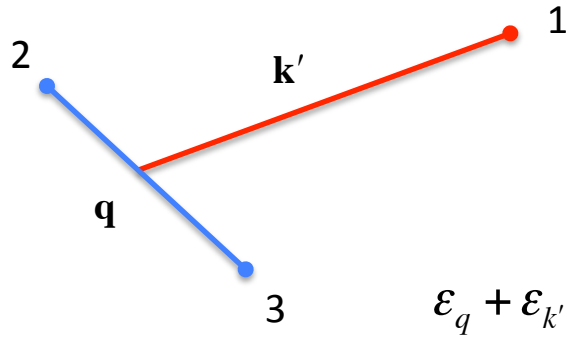








Three-Body Resonance Model



$\varepsilon_q + \varepsilon_{k'} = \varepsilon =$ total c.m. energy

$$T^{(3)}_{\mathbf{q}\mathbf{k}'\mathbf{k}} = \sum_{\lambda} c_\lambda(\mathbf{q}) \tilde{T}^{(2)}_{\lambda\mathbf{k}'\mathbf{k}},$$

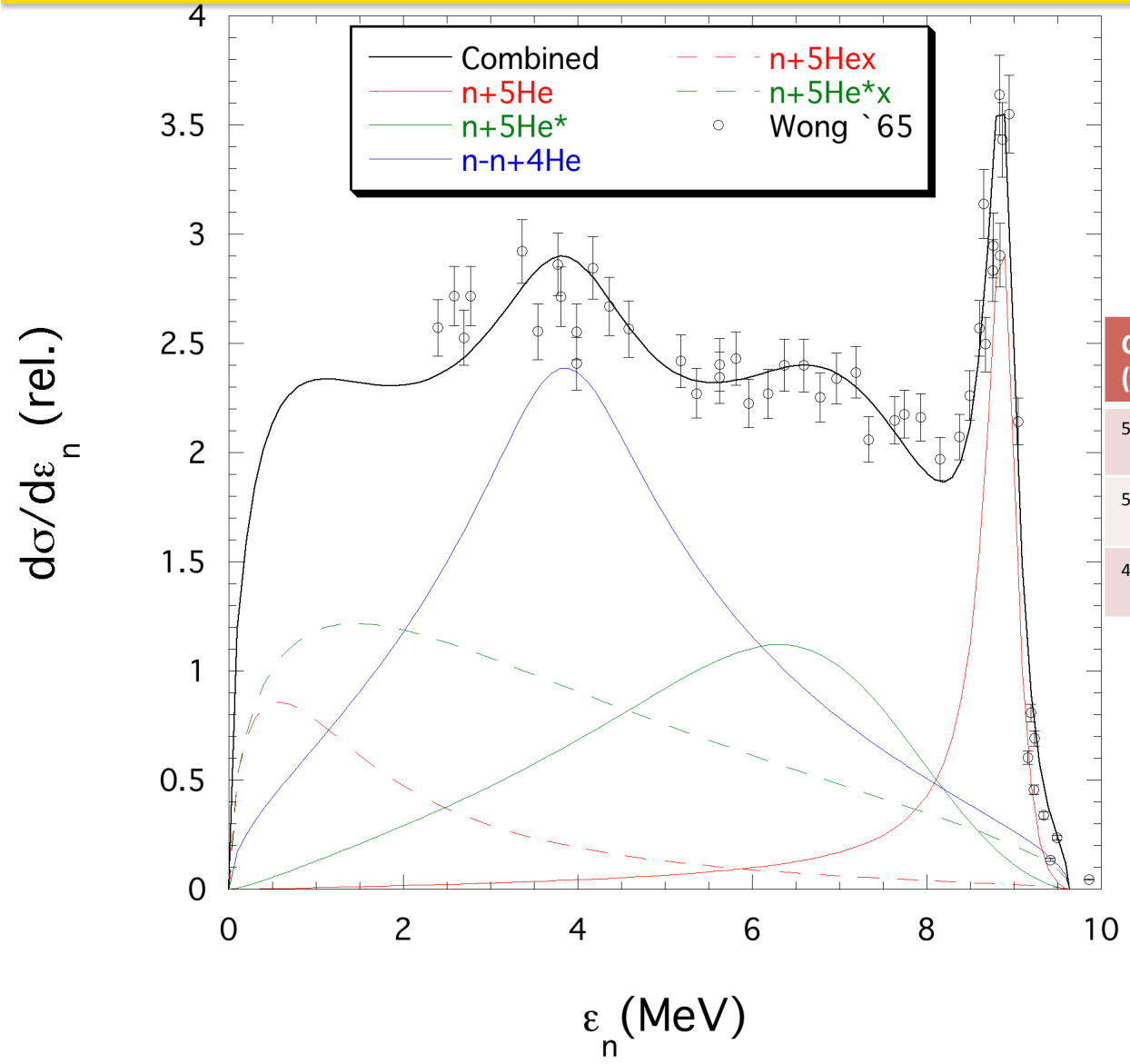
$$c_\lambda(\mathbf{q}) = \sqrt{\frac{\hbar^2 \Gamma_\lambda(\varepsilon_q)}{2\pi\mu_{23} q}} \left[\varepsilon_\lambda + \Delta_\lambda(\varepsilon_q) - (\varepsilon_q) - \frac{i}{2}\Gamma_\lambda(\varepsilon_q)\right]^{-1} \mathrm{e}^{-i\phi_{\tilde{l}}}\, Y^0_{\tilde{l}}(\hat{\mathbf{q}}),$$

$$\tilde{T}^{(2)}_{\lambda\mathbf{k}'\mathbf{k}} = \sum_{Js'l'sl} \mathcal{Y}_{\lambda Js'l'}(\hat{\mathbf{k}}') O^{-1}_{l'}(k') R^{\bar{L}}_{\lambda s'l'sl}(\varepsilon) O^{-1}_{l}(k) \mathcal{Y}^{*}_{Jsl}(\hat{\mathbf{k}}).$$

$$\bar{L}_{l'}(\varepsilon) = \frac{\gamma^2_\lambda}{\pi} \int_0^\varepsilon d\varepsilon_q L_{l'}(\varepsilon - \varepsilon_q) \frac{P_{\tilde{l}}(\varepsilon_q)}{\left|\varepsilon_\lambda - \varepsilon_q - \gamma^2_\lambda (L_{\tilde{l}}(\varepsilon_q) - \varepsilon)\right|^2}$$

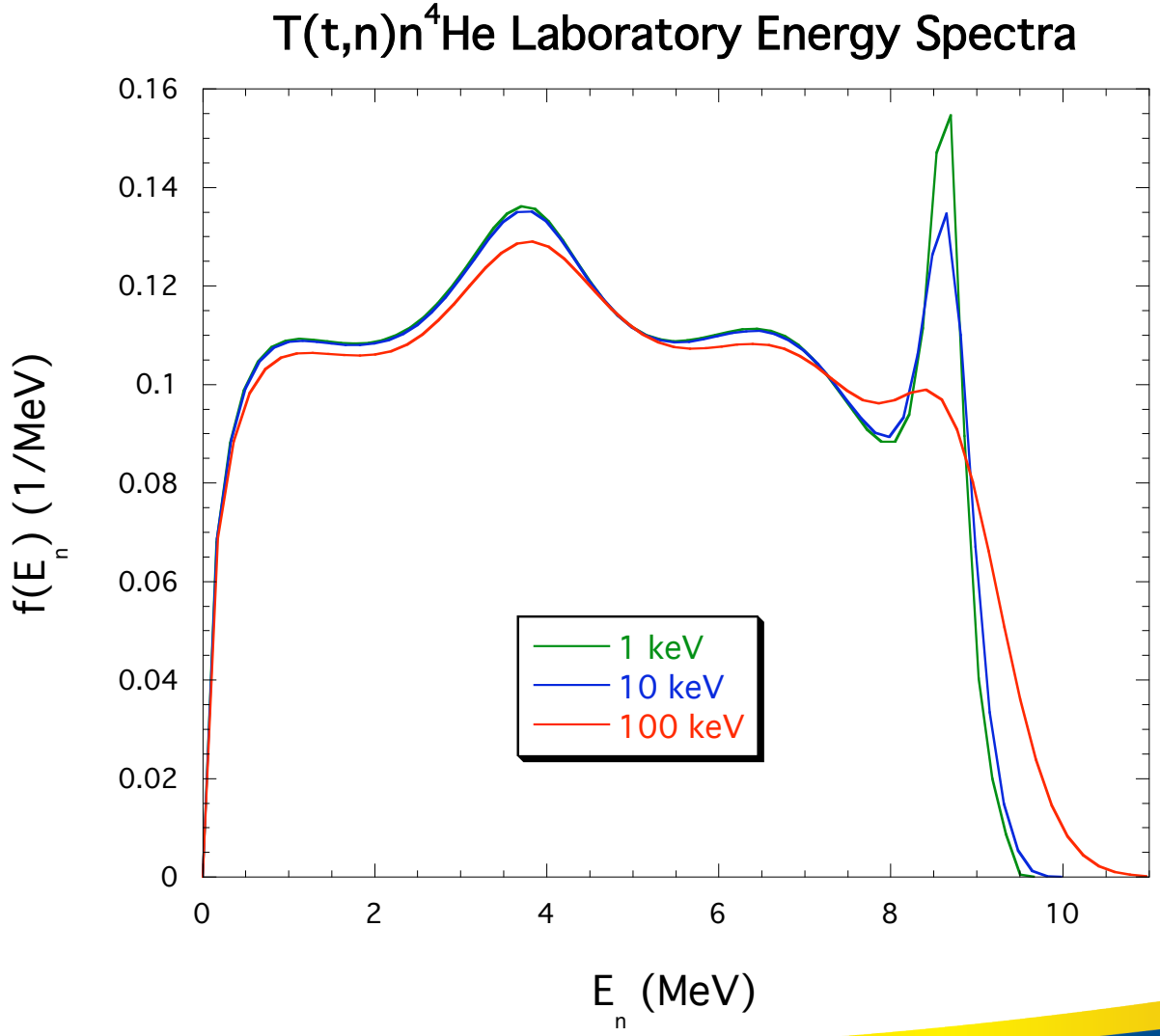
$$\frac{d^3\sigma}{d\mathbf{k}'} \propto \int d\mathbf{q} \left|T^{(3)}_{\mathbf{q}\mathbf{k}'\mathbf{k}}\right|^2 \delta(\varepsilon_q + \varepsilon_{k'} - \varepsilon)$$

T(t,n)nα c.m. Spectrum at E_t=0.5 MeV

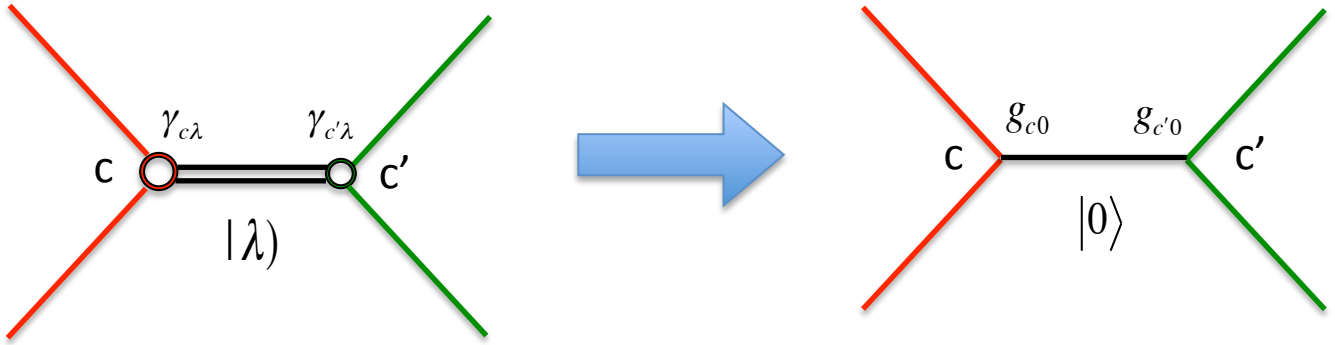


Channel (resonance)	J^π	E_R (MeV)	Γ (MeV)	$\lvert R^L \rvert$
^{5}He+n	$3/2^-$	0.99	0.96	0.182
^{5}He*+n	$1/2^-$	6.66	20.6	0.540
^{4}He+(n-n)	0^+	-0.07	0.	0.340

Calculated Maxwellian Averaged Spectra



R-Matrix and EFT

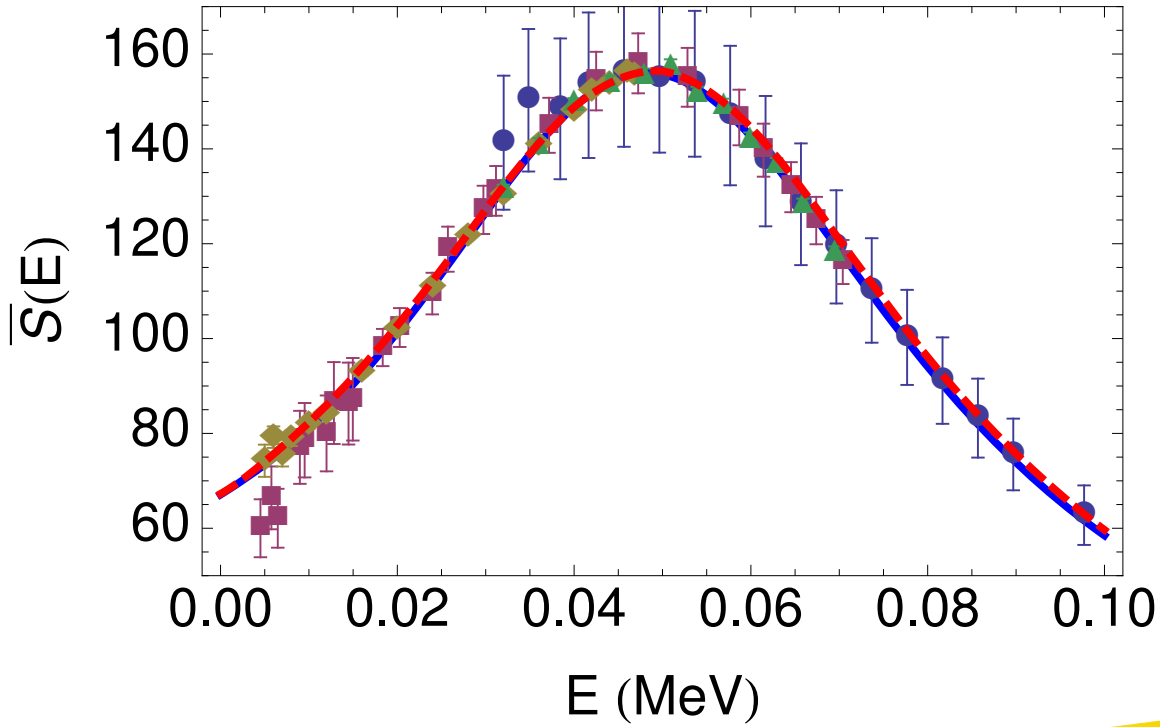


EFT Description of the T(d,n) Resonance

(with Lowell Brown)

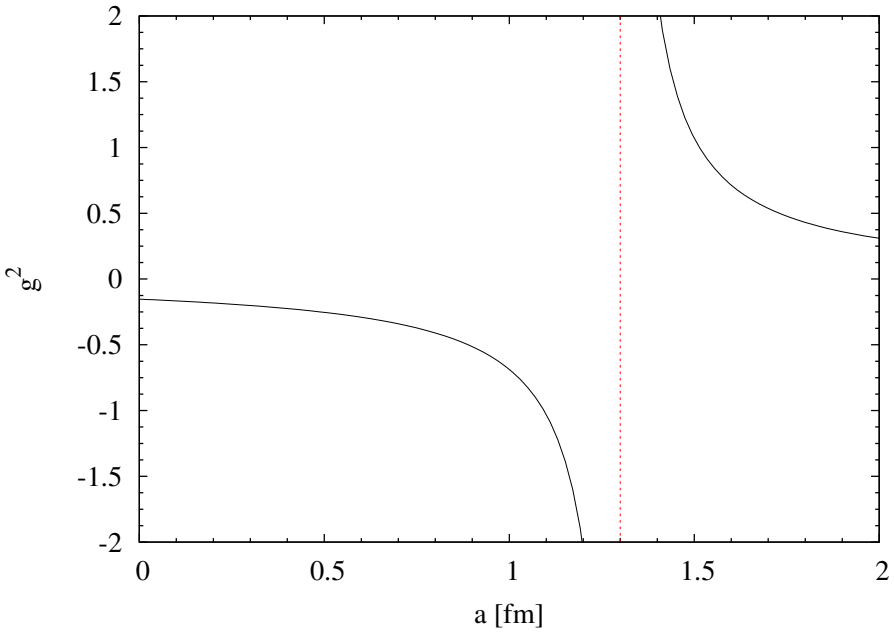
$$\bar{S}(E) = \frac{k_d^2}{D(\eta)}\sigma_{d,n}(E) = \frac{4}{9}\frac{g_d^2\mu_d}{\hbar^2 b_0}\frac{g_n^2\mu_n}{\hbar^2}k_n^5\left|E_0 + \Delta(\eta) - E + i\left[\frac{g_d^2\mu_d}{\hbar^2 b_0}D(\eta) + \frac{g_n^2\mu_n}{6\pi\hbar^2}k_n^5\right]\right|^{-2},$$

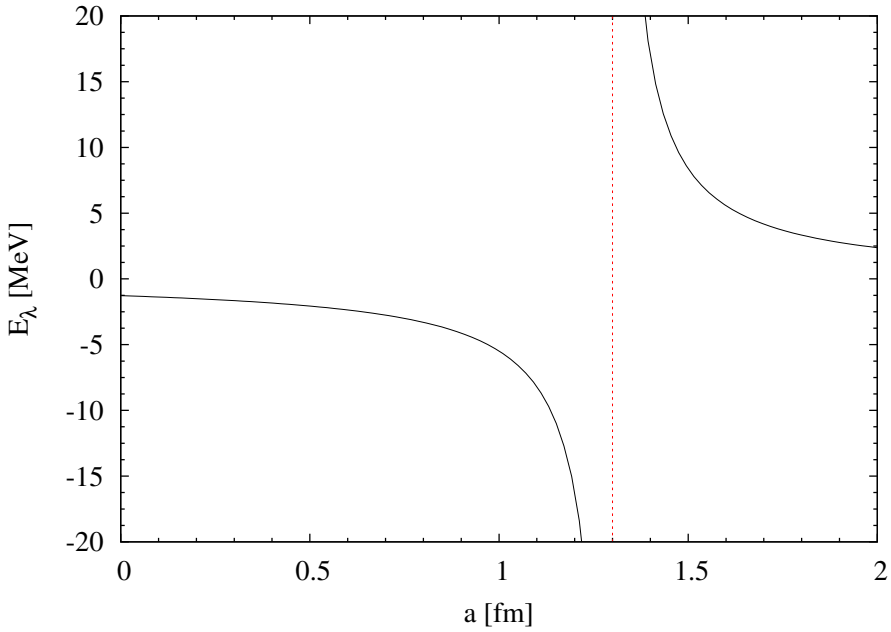
$$\Delta(\eta) = \frac{g_d^2\mu_d}{\pi\hbar^2 b_0}\left[\Re\psi(i\eta) - \ln(\eta)\right],\ D(\eta) = \left[\exp(2\pi\eta) - 1\right]^{-1}$$



SL R-matrix Parameters at Small Radii

For singlet *n*-*p* scattering, we investigated the behavior of $g^2 = a\gamma_\lambda^2 / \hbar c$ and E_λ as a function of a for fixed values of a_0 = -23.7 fm and r_0 = 2.75 fm when they are obtained from $R(E) = \gamma_\lambda^2 / (E_\lambda - E)$:





The same sort of pole behavior in the *R*-matrix parameters was observed for triplet *n*-*p* scattering and for the *d*+*t* reaction as the channel radii were varied.

Summary and Conclusions

- R-matrix theory contains all the basic requirements (unitarity, causality, and reciprocity) of the multi-channel scattering matrix in a simple pole expansion. It is therefore an extremely flexible phenomenological parametrization of experimental data that has the correct continuation to complex energies (momenta).
- The presence of channel radii in the theory has been criticized by its detractors, but they are useful measures of the sizes of the interacting particles and the ranges of the strong forces between them. Asymptotic quantities in the theory (e.g., *T*) are independent of channel radii in principle, and their poles and residues do not depend on them in practice.
- Recent work shows that, although there are minimum channel radii at which R-matrix parameters are physical, they can be taken even to zero if the reduced width amplitudes are allowed to be pure imaginary, thereby establishing a connection with "wrong-sign" Lagrangians in effective field theory.
- Quasi-stationary states may not be so useful for interpreting the behavior of time-dependent wave functions, especially for broad resonances.